

**Pembebanan Pada Struktur Beton Bertulang
Dan
Analisis Balok Lentur**

Ir. Sarjono Puro, MT

JENIS PEMBEBANAN

Jenis beban:

Beban mati/Dead Loads (DL) : berat sendiri struktur, beban permanen

Beban hidup/Live Loads (LL) : berubah besar dan lokasinya

Beban lingkungan : gempa (E), angin (W), hujan (R), dll

Kombinasi beban ditentukan oleh peraturan, misal:

1.4 D

1.2 D + 1.6 L

Analisis Lentur Balok Beton Bertulang

Balok mengalami 3 tahap sebelum runtuh:

1. Sebelum retak (*uncracked concrete stage*)
2. Setelah retak – tegangan elastis (*concrete cracked-elastic stresses stage*),
3. Kekuatan ultimate (*ultimate strength stage*)

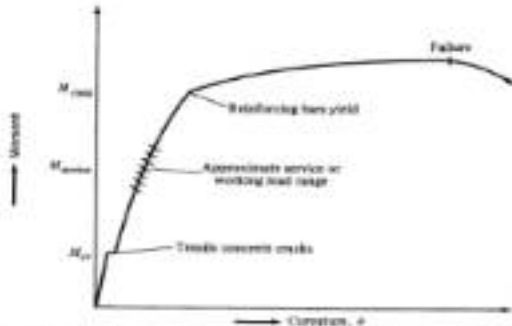
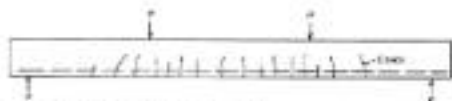


Figure 2.4 Moment-curvature diagram for reinforced concrete beam with internal reinforcing only.

Analisis Lentur Balok Beton Bertulang



(a) Stage A (before cracking)



(b) Stage B (after cracking, before yield)

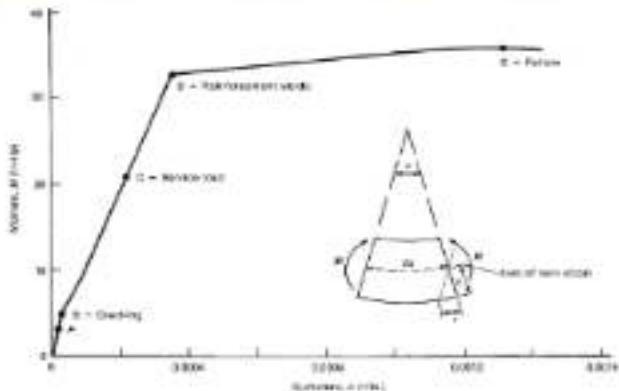


(c) Stage C (after yield)



Figure 1-5
Cracks, yielding, and ultimate flexure.

Analisis Lentur Balok Beton Bertulang



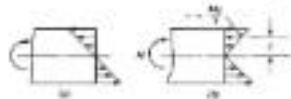
Analisis Lentur Balok Beton Bertulang

$$M = Cjd$$

$$M = \sigma_{s(max)} \frac{bd^2(2k)}{4 \left(\frac{3}{2} \right)}$$

$$M = \sigma_{s(max)} \frac{bd^2}{k/2}$$

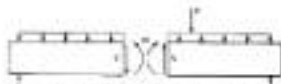
$$M = \frac{\sigma f}{y}$$



(e) Crack.



(f) Beam cross-section.



(g) Beam cross-section showing diagonal crack and shear force.



(h) Two beam cross-sections showing diagonal crack and shear force.

Uncracked concrete stage

- Tegangan tarik beton $f_t < f_r$
 - $f_t = 0.7 \sqrt{f_c'}$ (SI Unit)
 - $f_t = 7.5 \sqrt{f_c'}$ (US Unit)
- Dibatasi oleh momen pada saat retak (*cracking moment*) M_{cr}

$$M_{cr} = f_r I_g / y_t$$

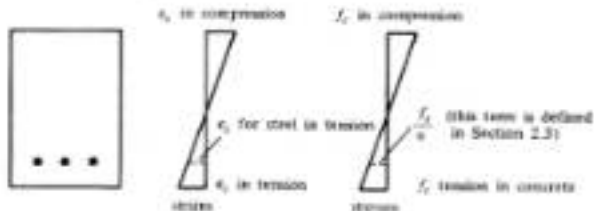


Figure 2.1 Uncracked concrete stage.

Contoh 1: Cracking Moment

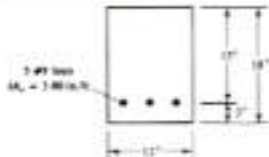


Figure 2.5

EXAMPLE 2.1

- (a) Assuming the concrete is uncracked, compute the bending stresses in the extreme fibers of the beam of Figure 2.5 for a bending moment of 25 ft-k. The concrete has an f'_c of 4000 psi and a modulus of rupture $f_r = 7.5\sqrt{4000} = 474$ psi.
- (b) Determine the cracking moment of the section.

SOLUTION

- (a) Bending stresses:

$$I_x = \frac{1}{12} bh^3 \text{ with } b = 12 \text{ in. and } h = 18 \text{ in.}$$

$$I_x = \left(\frac{1}{12} \right) (12)(18)^3 = 5832 \text{ in.}^4$$

$$f = \frac{My}{I_x} \text{ with } M = 25 \text{ ft-k} = 25,000 \text{ ft-lb}$$

Contoh 1: Cracking Moment

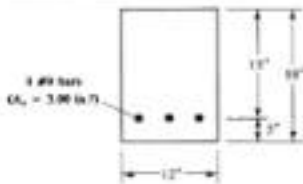


Figure 2.5

Then multiply the 25,000 by 12 to obtain 300,000 as shown below:

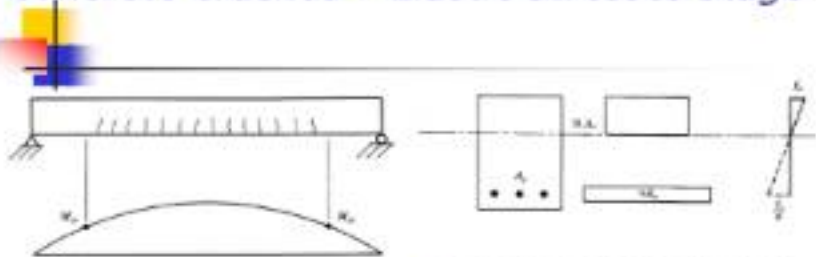
$$f = \frac{(12 \times 25,000)(3.00)}{5832} = 463 \text{ psi}$$

Since this stress is less than the tensile strength or modulus of rupture of the concrete of 474 psi, the section is assumed not to have cracked.

(b) Cracking moment:

$$M_{cr} = \frac{f_r I_g}{y} = \frac{(474)(5832)}{9.00} = 307,152 \text{ in.-lb} = \underline{25.6 \text{ ft.-k}}$$

Concrete Cracked – Elastic Stresses Stage



(a)

- Beton di bawah garis netral (NA) tidak memikul gaya tarik, dan sepenuhnya ditahan oleh baja
- NA ditentukan dengan prinsip *transformed area* ($n \times A_c$)
- Rasio modulus:

$$n = E_s/E_c$$



(b)



(c)

Figure 2.2 Concrete cracked—elastic stresses stage.

Contoh 2: Bending Moment for Cracked Concrete

EXAMPLE 2.2

Calculate the bending stresses in the beam shown in Figure 2.7 by using the transformed area method $n = 9$ and $M = 75 \text{ k-ft}$.

SOLUTION

Taking Moments about Neutral Axis (Referring to Figure 2.8)

$$(12n) \left(\frac{h}{2} \right) = (9)(3.00)(17) + (x)$$

$$9x^2 = 450 + 27.00x$$

Solving by Completing the Square

$$9x^2 + 27.00x - 450 = 0$$

$$x^2 + 3.00x - 50.0 = 0$$

$$(x + 1.25)(x + 3.25) = (3.25 + 1.25)^2$$

$$x = 2.25 \pm \sqrt{56.25} = (2.25) \pm 7.50$$

$$x = 9.75"$$



Figure 2.7



Figure 2.8

Moment of Inertia

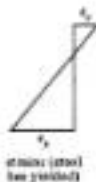
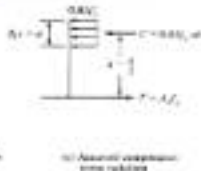
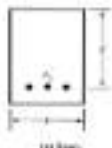
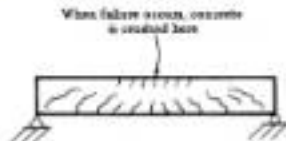
$$I = (12)(105.70)^2 + (9)(3.00)(14.22)^2 = 4067 \text{ in.}^4$$

Bending Stresses

$$f_c = \frac{My}{I} = \frac{(12)(75.00)(9.75)}{4067} = 2.09 \text{ ksi}$$

$$f_s = \frac{My}{I} = (9) \frac{(12)(75.00)(14.22)}{4067} = 13.00 \text{ ksi}$$

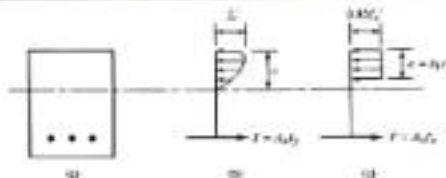
Ultimate Strength Stage



Asumsi:

- Tulangan tarik leleh sebelum beton di daerah tekan hancur
- Diagram kurva tegangan beton dapat didekati dengan bentuk segi empat

Ultimate Strength Stage



Penyederhanaan kurva tegangan beton:

- US Unit

For concretes with $f'_c > 4000$ psi, β_1 can be determined with the following formula:

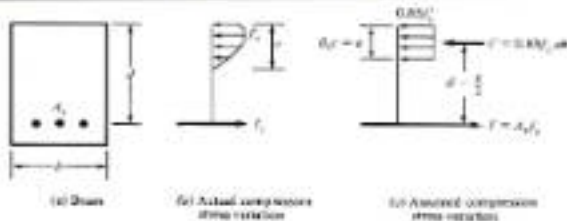
$$\beta_1 = 0.85 - \left(\frac{f'_c - 4000}{1000} \right) (0.05) \geq 0.65$$

- SI Unit

For concretes with $f'_c > 30$ MPa, β_1 can be determined with the following expression:

$$\beta_1 = 0.85 - 0.008(f'_c - 30) \geq 0.65$$

Ultimate Strength Stage



Prosedur Analisis:

1. Hitung gaya tarik $T = A_s f_y$
2. Hitung $C = 0.85 f_c' a b$, dan dengan $T = C$, tentukan nilai a
3. Hitung jarak antara T dan C (untuk penampang segi empat, jarak tersebut adalah $d - a/2$)
4. Tentukan M_n sebagai T atau C dikalikan dengan jarak antara kedua gaya tersebut

Contoh 3: Nominal moment

EXAMPLE 2.8

Determine M_n , the nominal or theoretical ultimate moment strength of the beam section shown in Figure 2.12 if $f'_c = 60,000 \text{ psi}$ and $f_y = 70,000 \text{ psi}$.

SOLUTION

Computing Tension and Compressive Forces T and C

$$T = A_s f_y = (3.00)(70) = 210 \text{ k}$$

$$C = 0.85 f'_c a b = (0.85)(60)(18)(a) = 91.8a$$

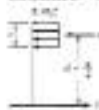
Equating T and C and Solving for a

$$T = C \text{ for equilibrium}$$

$$210 = 91.8a$$

$$a = 2.29 \text{ in.}$$

Computing $d = \frac{h}{2}$ and M_n



$$d = \frac{h}{2} = 21 = \frac{24}{2} = 12 \text{ in.}$$

$$M_n = (210)(21.0) = 4410 \text{ k-in.} = \frac{4410}{12} \text{ k-ft} = 367.5 \text{ k-ft}$$





Keruntuhan Balok Beton Bertulang

- Tension failure
 - tulangan leleh sebelum beton hancur
 - balok bersifat under-reinforced
- Compression failure
 - beton hancur sebelum tulangan leleh
 - balok bersifat over-reinforced
- Balanced failure
 - beton hancur dan tulangan leleh secara bersamaan
 - balok bersifat balanced-reinforced

Keruntuhan Balok Beton Bertulang



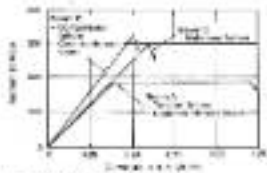
g) Beton s.d. Tension Cracks



h) Beton s.d. Tension Cracks



i) Beton s.d. Tension Cracks



j) Beton s.d. Tension Cracks



Luas Tulangan Minimum

- Diperlukan untuk mencegah balok runtuh mendadak
- Berdasarkan peraturan:

$$A_{smin} = \frac{2\sqrt{f'_c}}{f_y} b_s d \quad (\text{ACI Equation 10-3})$$

$$\text{or less than } \frac{200 b_s d}{f_y}$$

[In SI units, these expressions are $\left(\frac{\sqrt{f'_c}}{4f_y}\right) b_s d$ and $\left(\frac{1.40 b_s d}{f_y}\right)$, respectively.]



Luas Tulangan *Balanced* ρ_b

- Beton hancur dan tulangan leleh secara bersamaan

$$\rho_b = \left(\frac{0.85\beta_1 f'_c}{f_y} \right) \left(\frac{87,000}{87,000 + f_y} \right)$$

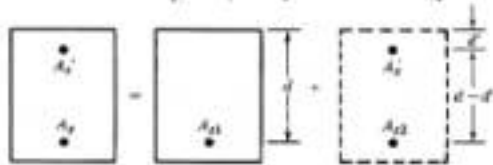
$$\text{or in SI units } \left(\frac{0.85\beta_1 f'_c}{f_y} \right) \left(\frac{600}{600 + f_y} \right)$$

Tulangan Tekan/Negatif

- Tulangan tekan/negatif adalah tulangan yang berada di daerah tekan balok
- Balok yang mempunyai tulangan tarik dan tekan disebut doubly reinforced beams
- Momen Nominal:

$$M_c = A_{s1}f_y \left(d - \frac{a}{2} \right) + A_{s2}f_y (d - d')$$

$$\phi M_o = \phi \left[A_{s1}f_y \left(d - \frac{a}{2} \right) + A_{s2}f_y (d - d') \right]$$



$$M_o = M_{o1} + M_{o2} \quad M_{o1} = A_{s1}f_y \left(d - \frac{a}{2} \right)$$

$$M_{o2} = A_s'f_y (d - d') = A_{s2}f_y (d - d')$$

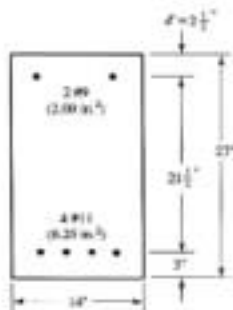
(a)

(b)

(c)

Contoh 4: Doubly Reinforced Beams

Determine the design moment capacity of the beam shown in Figure 5.14 for which $f_c' = 40,000$ psi and $f_y = 3000$ psi.



SOLUTION

Writing the Quadratic Equation for Equilibrium

$$(6.25)(60) = (0.85)(3)(14)(0.85c) + (2.00)\left(\frac{c - 2.5}{c}\right)(3000)(29,000)$$

Multiplying Through by c and Simplifying

$$c^2 - 6.624c - 14.335$$

Solving Equation

$$c = 8.34 \text{ in.}$$

$$A_s = (0.85)(8.34) = 7.09 \text{ in.}$$

Figure 5.14

Contoh 4: Doubly Reinforced Beams

Computing Strain in Compression and Tensile Steel

$$\epsilon_s' = \left(\frac{8.54 - 7.00}{8.54} \right) (0.005) = 0.00210$$
$$> \frac{60}{29,000} = 0.00207 \quad \therefore f_s' = 60 \text{ ksi}$$

$$A_s f_s = A_s' f_s'$$

$$A_s = \frac{A_s' f_s'}{f_s} = \frac{(2.00)(60)}{60} = 2.00 \text{ in}^2$$

$$A_s = A_s - A_s' = 6.25 - 2.00 = 4.25 \text{ in}^2$$

$$\epsilon_s = \left(\frac{24 - 8.54}{8.54} \right) (0.003) = 0.00563 > 0.005$$
$$\phi = 0.90$$

Then the Design Moment Strength Is

$$\phi M_n = \phi \left[A_s f_s \left(d - \frac{a}{2} \right) + A_s' f_s' (d - d') \right]$$
$$= 0.90 \left[(4.25)(60) \left(24 - \frac{7.00}{2} \right) + (2.00)(60)(21.5) \right]$$
$$= 5038 \text{ in.-k} = \underline{384.7 \text{ ft.-k}}$$

Tulangan Transversal/Geser

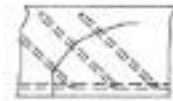
- Memikul sebagian gaya geser pada balok
- Menahan retak geser pada balok
- Meningkatkan kekuatan dan daktilitas balok



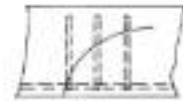
(a) Retakan geser



(b) Retakan geser

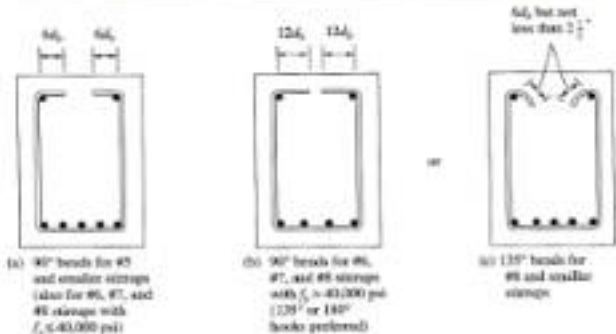


(c) Retakan geser menahan geser



(d) Retakan geser menahan geser

Tulangan Transversal (*Stirrup*)



Note: Put stirrups as close to compression and tension surfaces as cover and other reinforcing permits.

Figure 8.7 Stirrup details.



Kekuatan Geser Balok

- Kuat geser nominal:

- $V_n = V_c + V_s$

- Kuat geser beton:

$$V_c = 2 \sqrt{f_c'} b_w d \quad (\text{US Unit})$$

$$V_c = (\sqrt{f_c'} b_w d)/6 \quad (\text{SI Unit})$$

- Kuat geser tulangan:

$$V_s = A_v f_y d/s$$



Table 8.1 Summary of Steps Involved in Vertical Stirrup Design

Is shear reinforcing necessary?
1. Draw V_u diagram. 2. Calculate V_u at a distance d from the support (with certain exceptions). 3. Calculate $\phi V_c = 2\phi\sqrt{f'_c}b_v d$ (or use the alternate method). 4. Stirrups are needed if $V_u > \phi V_c$ (with some exceptions for slabs, footings, shallow members, and joints).
Design of stirrups
1. Calculate theoretical stirrup spacing $s = A_v f_y / V_u$, where $V_u = (V_u - \phi V_c) / \phi$. 2. Determine maximum spacing to provide minimum area of shear reinforcement: $s = \frac{A_v f_y}{0.75\sqrt{f'_c}b_v}$ but not more than $\frac{A_v f_y}{50b_v}$ 3. Compute maximum spacing: $d/2 \leq 24$ in., if $V_u \leq 4\sqrt{f'_c}b_v d$. 4. Compute maximum spacing: $\frac{d}{4} \leq 12$ in., if $V_u > 4\sqrt{f'_c}b_v d$. 5. V_u may not be $> 8\sqrt{f'_c}b_v d$. 6. Minimum practical spacing = approximately 3 or 4 in.

Contoh 5: Stirrup

The beam shown in Figure 8.10 is 40 in. wide and using $f_c = 40,000$ psi and $f_s = 60,000$ psi. Determine the theoretical spacing of #3 stirrups for each of the following loads.

- a) $V_u = 12,000$ lb
- b) $V_u = 16,000$ lb
- c) $V_u = 40,000$ lb
- d) $V_u = 75,000$ lb

ANSWER

- a) $V_u = 12,000$ lb

$$\phi V_c = 1.1 \sqrt{f_c} b_w d = 1.1 \sqrt{40,000} (40)(27) = 77,424 \text{ lb}$$

$$\phi V_s = 12,000 \text{ lb} < 77,424 \text{ lb}$$

→ No stirrups required

$$V_u = 16,000 \text{ lb}$$

Stirrups needed because $V_u > \phi V_c$.
Theoretical spacing:

$$\phi V_s = 40,000 - 77,424$$

$$s = \frac{A_v f_s}{\phi V_s} = \frac{3(0.11)(60,000)}{1.1(40,000 - 77,424)} = 0.79$$

$$s = \frac{A_v f_s}{\phi V_s} = \frac{3(0.11)(60,000)(16)}{1.1(40,000)} = 2.00 \text{ ft} = 24 \text{ in.}$$

Maximum spacing is provided in Table 4:

$$s = \frac{A_v f_s}{\phi V_s} = \frac{3(0.11)(60,000)}{1.1(40,000 - 77,424)} = 0.79 \text{ ft}$$

$$s = \frac{A_v f_s}{\phi V_s} = \frac{3(0.11)(60,000)(16)}{1.1(40,000)} = 2.00 \text{ ft}$$

$$s = 75,000 \times (40)(27)(14)(24) = 75,000$$

$$s = 11.52 \text{ in.}$$

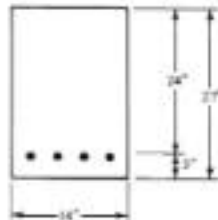


Figure 8.10

Contoh 5: Stirrup

- (a) $V_u = 60,000$ lb
Theoretical spacing

$$V_u = \frac{V_c + \phi V_s}{\phi} = \frac{60,000 - 11,609}{0.75} = 41,593 \text{ lb}$$

$$s = \frac{A_s f_y}{V_u} = \frac{(2)(0.11)(60,000)(174)}{41,593} = 7.37 \text{ in}$$

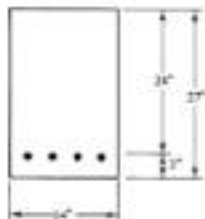
Maximum spacing to provide minimum s

$$s = \frac{A_s f_y}{0.75 \sqrt{f'_c} b_v} = \frac{(2)(0.11)(60,000)(174)}{0.75 \sqrt{3000} (14)} = 22.97 \text{ in}$$

$$s = \frac{A_s f_y}{36 b_v} = \frac{(2)(0.11)(60,000)(174)}{(36)(14)} = 18.86 \text{ in}$$

$$V_c = 41,593 < 100 \sqrt{3000} (14)(24) = 71,614 \text{ lb}$$

Maximum $s = \frac{d}{3} = 17 \text{ in}$ $s = 7.37 \text{ in}$



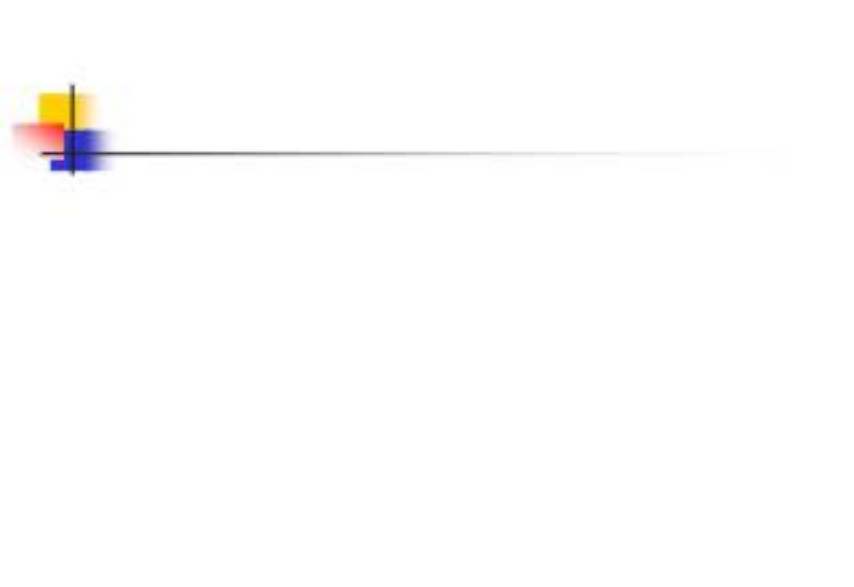
- (b) $V_u = 150,000$ lb

$$V_u = \frac{150,000 - 27,687}{0.75} = 103,195 \text{ lb}$$

$$103,195 \text{ lb} > 100 \sqrt{3000} (14)(24) = 71,614 \text{ lb}$$

V_u may not be taken $> 8 \sqrt{f'_c} b_v d$

$\therefore$ Must larger beam and/or one with larger f'_c value



Perencanaan Balok
(Komponen Struktur Lentur)
pada SNI



Komponen Struktur Lentur (Balok)

Persyaratan Gaya:

- Gaya aksial tekan terfaktor pada komponen struktur tidak melebihi $0,1A_g f_c$

Persyaratan Geometri:

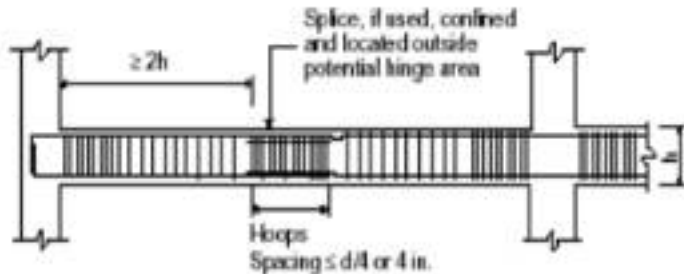
- Bentang bersih komponen struktur tidak boleh kurang dari empat kali tinggi efektifnya.
- Perbandingan lebar terhadap tinggi $\geq 0,3$.
- Lebar penampang haruslah
 - (a) ≥ 250 mm,
 - (b) $\leq$ lebar kolom ditambah jarak pada tiap sisi kolom yang tidak melebihi tiga perempat tinggi komponen struktur lentur



Persyaratan Tulangan Lentur

- Jumlah tulangan atas dan bawah tidak boleh kurang dari tulangan minimum atau $1,4b_w d/f_y$ dan rasio tulangan ρ tidak boleh melebihi 0,025. Harus ada minimum dua batang tulangan atas dan dua batang tulangan bawah yang dipasang secara menerus
- Kuat lentur positif balok pada muka kolom harus $\geq$ setengah kuat lentur negatifnya. Kuat lentur negatif dan positif pada setiap penampang di sepanjang bentang harus $\geq$ seperempat kuat lentur terbesar pada bentang tersebut.
- Sambungan lewatan pada tulangan lentur harus diberi tulangan spiral atau sengkang tertutup yang mengikat sambungan tersebut.
- Sambungan lewatan tidak boleh digunakan (a) pada daerah hubungan balok-kolom (b) pada daerah hingga jarak dua kali tinggi balok dari muka kolom, dan (c) pada tempat-tempat yang berdasarkan analisis, memperlihatkan kemungkinan terjadinya leleh lentur akibat perpindahan lateral inelastis struktur rangka

Persyaratan Sambungan Lewatan





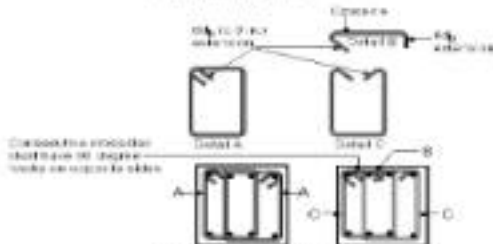
Persyaratan Tulangan Transversal

- Sengkang tertutup harus dipasang:
 - Pada daerah hingga dua kali tinggi balok diukur dari muka tumpuan
 - Di sepanjang daerah dua kali tinggi balok pada kedua sisi dari suatu penampang yang berpotensi membentuk sendi plastis
- Sengkang tertutup pertama harus dipasang tidak lebih dari 50 mm dari muka tumpuan. Spasi sengkang tertutup tidak boleh melebihi
 - (a) $d/4$,
 - (b) delapan kali diameter terkecil tulangan memanjang
 - (c) 24 kali diameter batang tulangan sengkang tertutup,dan
 - (d) 300 mm.

Tulangan Transversal Balok (Confinement/Kekangan)

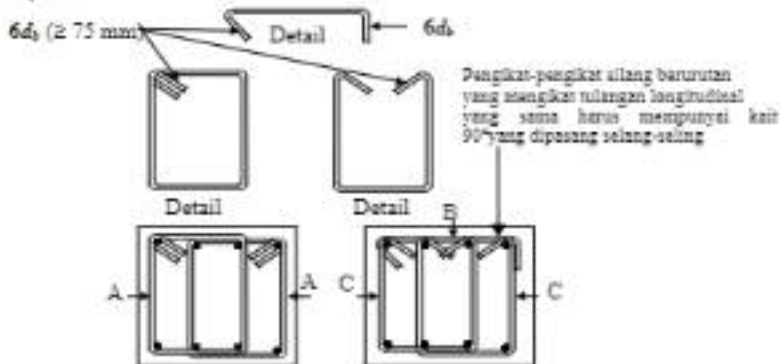


Typical Lap Joint Details



Single- and Two-Place Hooks

Contoh Sengking Tertutup yang Dipasang Bertumpuk





Persyaratan Kuat Geser

- Gaya Rencana

Gaya geser rencana V_u harus ditentukan dari peninjauan gaya statik pada bagian komponen struktur antara dua muka tumpuan

- Tulangan transversal

Tulangan transversal harus dirancang untuk memikul geser dengan menganggap $V_c = 0$ bila:

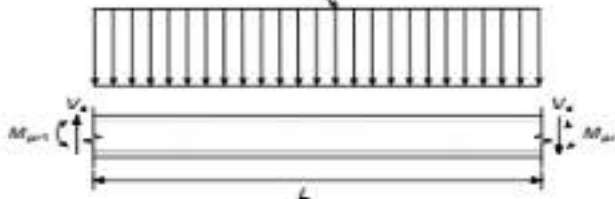
- a. Gaya geser akibat gempa mewakili setengah atau lebih daripada kuat geser perlu maksimum di sepanjang daerah tersebut, dan
- b. Gaya aksial tekan terfaktor, termasuk akibat gempa, lebih kecil dari $A_s f_s / 20$

Perencanaan Geser untuk Balok

Untuk balok:

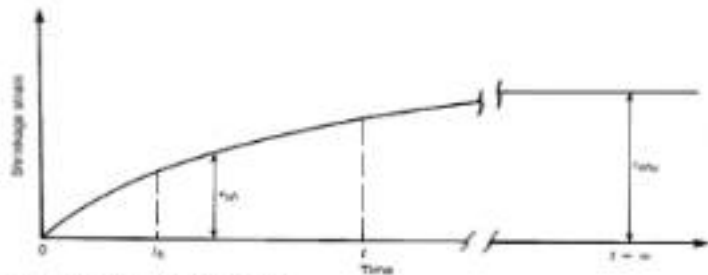
$$V_u = \frac{M_{u1} + M_{u2}}{L} \pm \frac{W_u L}{2}$$

Beban gravitasi $W_u = 1,2D + 1,0L$



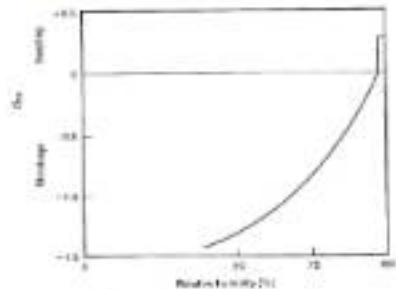
Momen ujung M_{pu} didasarkan pada tegangan tarik $1,25 f_y$

Susut

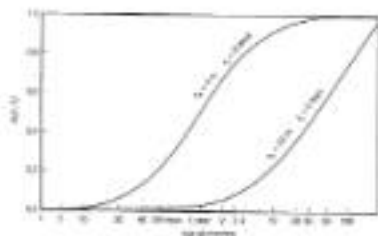


(a) Shrinkage of an unloaded specimen.

Susut

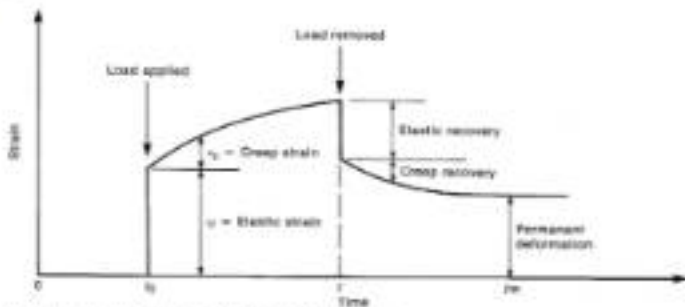


- Efek kelembaban pada susut



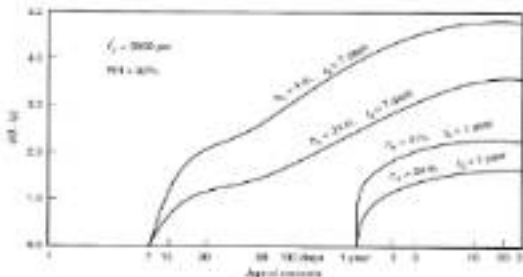
- Efek ketebalan beton pada susut

Rangkak



(b) Elastic and creep strains due to loading at time, t_0 , and unloading at time, t .

Rangkak



- Efek ketebalan beton pada rangkak